Dielectric Properties and Boundary Conditions

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Agenda

Intended Learning Outcomes

Dielectrics and Polarization

Dielectric Constant and Susceptibility

Boundary Conditions for Dielectrics

Capacitance

Multiple-Dielectric Capacitors

Intended Learning Outcomes

In this chapter, we consider insulating materials, or dielectrics.

Such materials differ from conductors in that ideally, there is no free charge that can be transported within them to produce conduction current.

An applied electric field has the effect of displacing the charges slightly, leading to the formation of ensembles of electric dipoles; this process is called Polarization . The extent to which this occurs is measured by the relative permittivity, or dielectric constant.

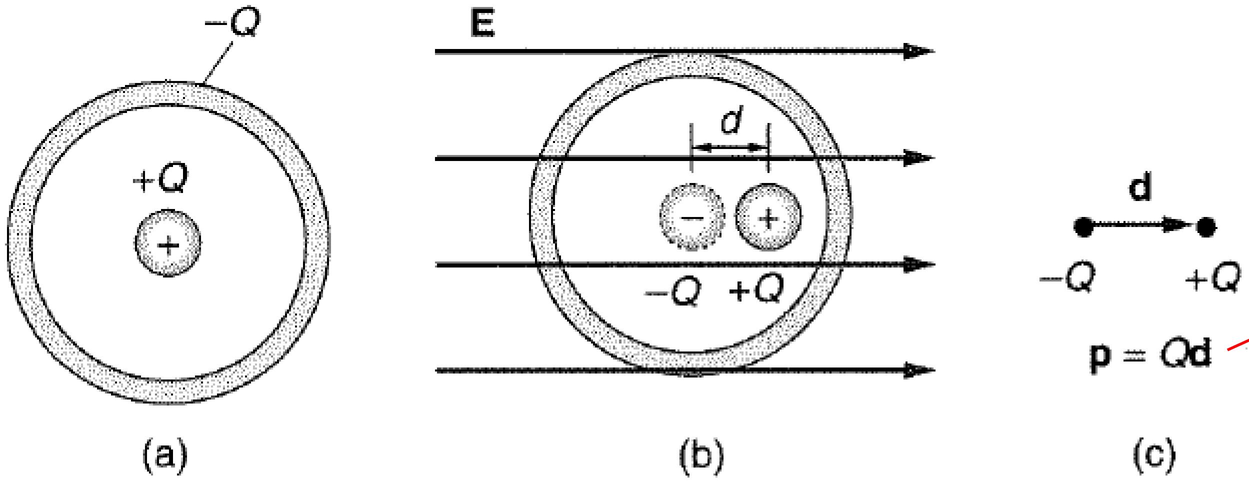
Boundary conditions for the fields at interfaces between dielectrics are developed to evaluate these differences.

In the end, the methods for calculating capacitance for a number of cases are presented, including transmission line geometries, and to be able to make judgments on how capacitance will be altered by changes in materials or their configuration.

Dielectrics and Polarization

- Dielectrics are materials which have no free charges; all electrons are bound and associated with the nearest atoms (Fig. 7.1(a)). They behave as electrically neutral when they are not in an electric field.
- An external applied electric field causes microscopic separations of the centers of positive and negative charges as shown in Fig 7.1 (b).
- These separations behave like electric dipoles, and this phenomenon is known as dielectric polarization.

Dielectrics and Polarization



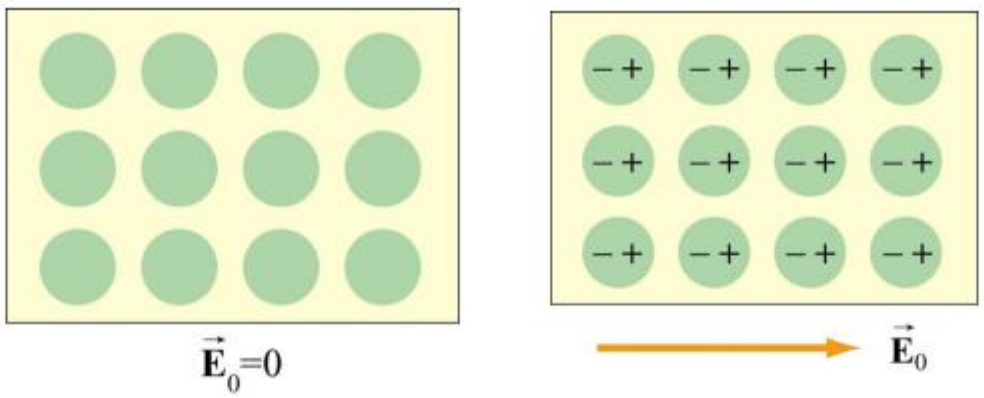
Electric dipole moment

Figure 7.1 (a) Model of a nonpolar molecule, (b) the molecule in an external electric field, and (c) the electric dipole that produces the same field as the molecule in (b)

Dielectrics and Polarization

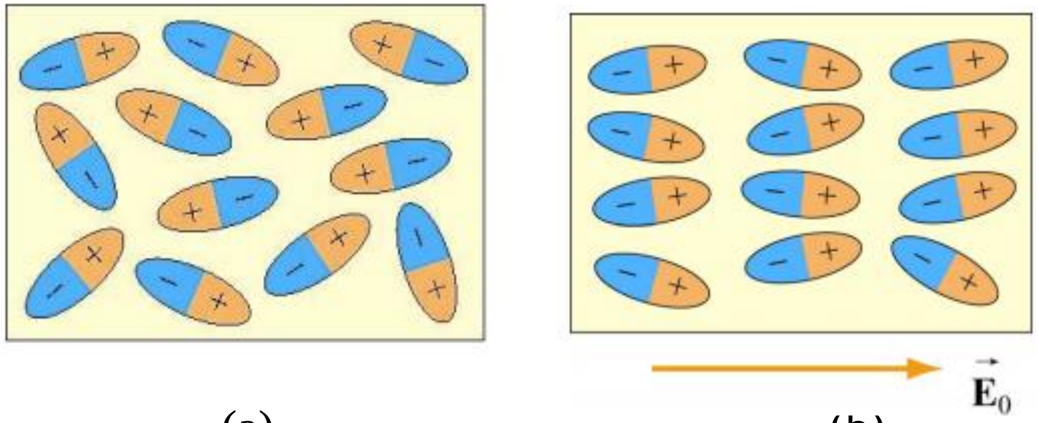
Dielectrics may be subdivided into two groups :

- **Non-Polar:** dielectrics that do not possess permanent electric dipole moment. Electric dipole moments can be induced by placing the materials in an externally applied electric field.
- **Polar:** dielectrics that possess permanent dipole moments which are ordinarily randomly oriented, but which become more or less oriented by the application of an external electric field. An example of this type of dielectric is water.



(a) (b)

Orientations of non-polar molecules when(a) $E_0 = 0$, and (b) $E_0 \neq 0$.



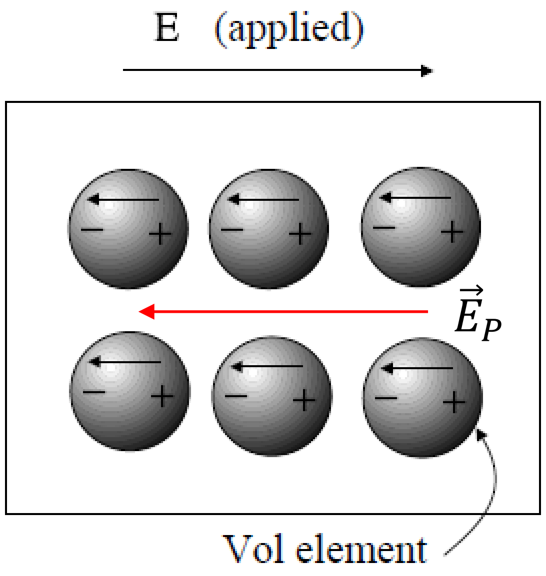
(a) (b)

Orientations of polar molecules when(a) $E_0 = 0$, and (b) $E_0 \neq 0$.

Dielectrics and Polarization

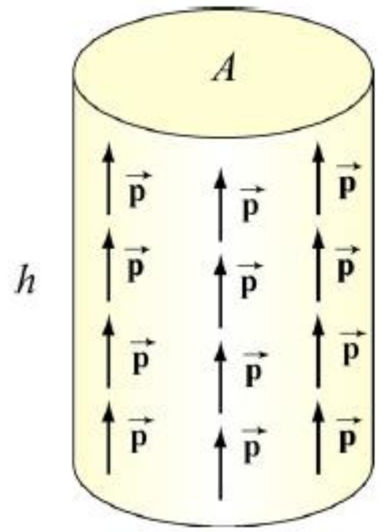
However, the alignment is not complete due to random thermal motion.

The aligned molecules (induced dipoles) then generate an electric field that is opposite to the applied field but smaller in magnitude.



Dielectrics and Polarization

Suppose we have a piece of material in the form of a cylinder with area A and height h, and that it consists of N electric dipoles, each with electric dipole moment $\vec{p}$ spread uniformly throughout the volume of the cylinder.



what is the average electric field just due to the presence of the aligned dipoles?

To answer this question, let us define the polarization vector $\vec{P}$ to be the net electric dipole moment vector per unit volume:

$$\vec{\mathbf{P}} = \frac{1}{\text{Volume}} \sum_{i=1}^{N} \vec{\mathbf{p}}_i \qquad (1)$$

Dielectrics and Polarization

In the case of our cylinder, where all the dipoles are perfectly aligned, the magnitude of $\vec{P}$ is equal to

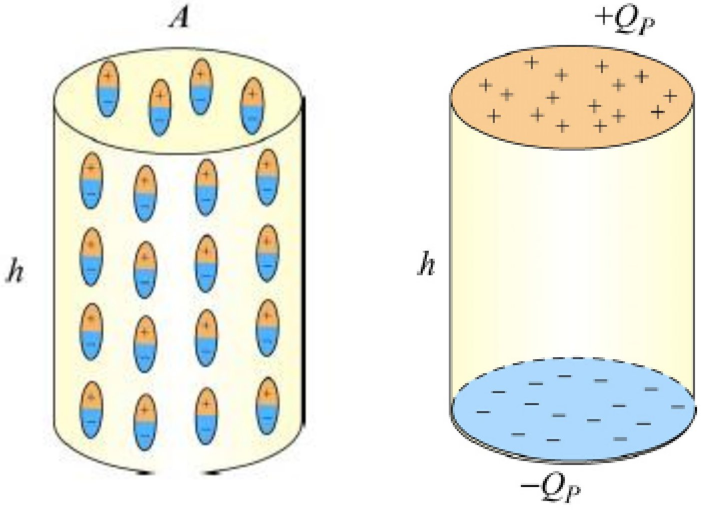
$$P = \frac{Np}{Ah} \qquad \text{C/m}^2$$

From the equivalence between (Fig. (a) and (b)), we have two net charges $\pm Q_P$ which produce net dipole moment of $Q_P h$; *hence*

$$Q_P = \frac{Np}{h}$$

we note that the equivalent charge distribution resembles that of a parallel-plate capacitor, with an equivalent surface charge density σ_P that is equal to the magnitude of the polarization:

$$\sigma_P = \frac{Q_P}{A} = \frac{Np}{Ah} = P$$



(a) (b)

(a) A cylinder with uniform dipole distribution.
(b) Equivalent charge distribution.

Dielectrics and Polarization

Thus, our equivalent charge system will produce an average electric field of magnitude $\vec{E}_P = \vec{P}/\epsilon_0$. Since the direction of this electric field is opposite to the direction of $\vec{P}$, in vector notation, we have

$$\vec{\mathbf{E}}_P = -\vec{\mathbf{P}} / \varepsilon_0 \qquad (2)$$

The total electric field $\vec{E}$ is the sum of these two fields:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 + \vec{\mathbf{E}}_P = \vec{\mathbf{E}}_0 - \vec{\mathbf{P}} / \varepsilon_0 \qquad (3)$$

In most cases, the polarization $\vec{P}$ is not only in the same direction as $\vec{E}$ but also linearly proportional to $\vec{E}_0$(and hence $\vec{E}$.) This is reasonable because without the external field there would be no alignment of dipoles and no polarization. We write the linear relation between and as $\vec{\mathbf{P}} = \varepsilon_0 \chi_e \vec{\mathbf{E}}$ (4)

Dielectrics and Polarization

where χ_e is called the electric susceptibility. Materials that obey this relation are linear dielectrics. Combing Eqs. (2) and (3) gives

$$\vec{\mathbf{E}}_0 = (1+\chi_e)\vec{\mathbf{E}} = \kappa_e\vec{\mathbf{E}} \qquad (5)$$

where

$$\varepsilon_r = \kappa_e = 1 + \chi_e$$

is the dielectric constant. The dielectric constant κ_e is always greater than one since $\chi_e > 0$.

Thus, we see that the effect of dielectric materials is always to decrease the electric field below what it would otherwise be.

Dielectric Constant and Susceptibility

Electric flux Density (D):

Electric flux density is defined as charge per unit area and it has same units of dielectric polarization.

$$D = \varepsilon_0 E + P = \varepsilon_0 (1+\chi_e) E$$

Electric flux density D at a point in a free space or air in terms of Electric field strength is

$$D_0 = \varepsilon_0 E$$

At the same point in a medium is given by $D = \varepsilon E$

As the polarization measures the additional flux density arising from the presence of material as compared to free space

$$D = \varepsilon_0 \varepsilon_r E \qquad (6)$$

Definitions

Polarization: the process of creating or inducing dipoles in a dielectric medium by an external field.

Polarizability: the ability of dielectric to form instantaneous dipoles. It is a property of matter

Polarization vector: it is defined as the dipole moment per unit volume.

$$\mathbf{P} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{i=1}^{n \Delta v} \mathbf{p}_i \qquad \text{C/m}^2$$

Electric susceptibility (χ_e) is a dimensionless proportionality constant that indicates the degree of polarization of a dielectric material in response to an applied electric field. The greater the electric susceptibility, the greater the ability of a material to polarize in response to the field, and thereby reduce the total electric field inside the material (and store energy).

$$\chi_e = \varepsilon_r - 1$$

Definitions

permittivity (ε): is the measure of resistance that is encountered when forming an electric field in a medium. In other words, permittivity is a measure of how an electric field affects, and is affected by, a dielectric medium.

$$\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi)\varepsilon_0$$

permittivity ε is measured in farads per meter, and ε_r is the relative permittivity of the material.

Dielectric Constant (relative permittivity) gives a measure of the polarizability of a material relative to free space and is defined as the ratio between the permittivity of the medium to the permittivity of free space.

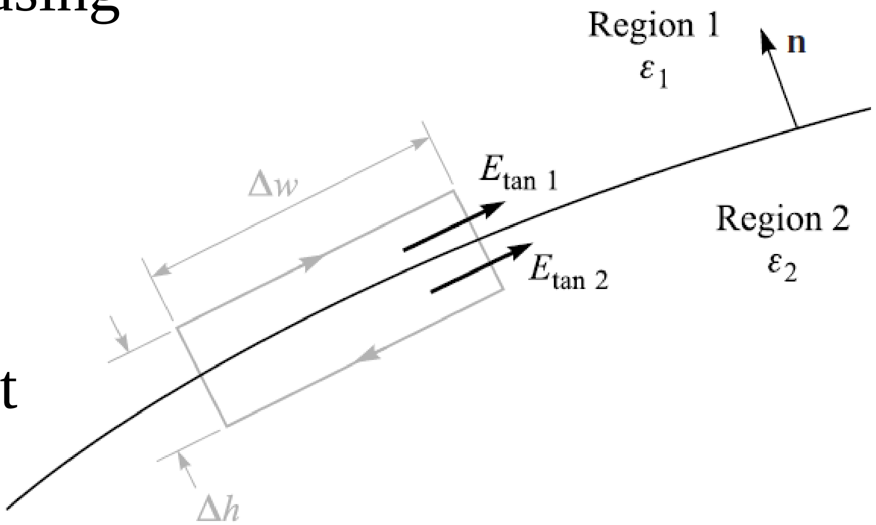
$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

Boundary conditions for Dielectric materials

Let us first consider the interface between two dielectrics having permittivities ε_1 and ε_2 and occupying regions 1 and 2, as shown in the figure below.

We first examine the tangential components by using

$$\oint \mathbf{E} \cdot d\mathbf{L} = 0$$



around the small closed path, obtaining

$$E_{\tan 1}\,\Delta w - E_{\tan 2}\,\Delta w = 0$$

The small contribution by the normal component of E along the sections of length Δh becomes negligible as Δh decreases and the closed path crowds the surface. Immediately, then,

$$E_{\tan 1} = E_{\tan 2} \qquad (7)$$

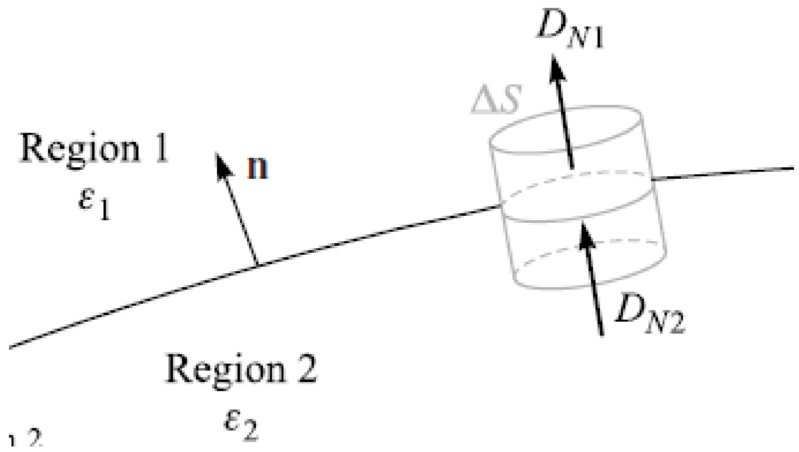
Boundary conditions for Dielectric materials

The boundary conditions on the normal components are found by applying Gauss's law to the small "pillbox" shown in the figure below.

The sides are again very short, and the flux leaving the top and bottom surfaces is the difference

$$D_{N1}\Delta S - D_{N2}\Delta S = \Delta Q = \rho_S \Delta S$$

$$D_{N1} - D_{N2} = \rho_S \qquad (8)$$



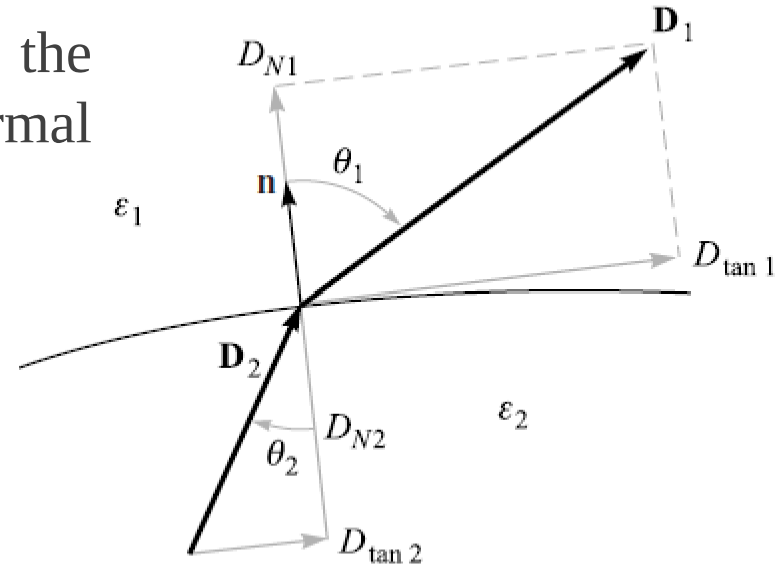
This charge may be placed there deliberately, thus unbalancing the total charge in and on this dielectric body. Except for this special case, ρ_S is zero on the interface; Hence

$$D_{N1} = D_{N2} \quad \text{or} \quad \epsilon_1 E_{N1} = \epsilon_2 E_{N2} \qquad (9)$$

Boundary conditions for Dielectric materials

Let D1 (and E1) make an angle θ_1 with a normal to the surface as shown in the figure. Because the normal components of D are continuous,

$$D_{N1} = D_1 \cos\theta_1 = D_2 \cos\theta_2 = D_{N2} \qquad (10)$$



But $$\frac{D_{\tan 1}}{\epsilon_1} = E_{\tan 1} = E_{\tan 2} = \frac{D_{\tan 2}}{\epsilon_2} \Rightarrow \frac{D_{\tan 1}}{D_{\tan 2}} = \frac{\epsilon_1}{\epsilon_2}$$

Thus $$\epsilon_2 D_1 \sin\theta_1 = \epsilon_1 D_2 \sin\theta_2$$

and the division of this equation by (10) gives

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2} \qquad (11)$$

Boundary conditions for Dielectric materials

Example: Find the fields within the Teflon slab (ε_r = 2.1), given the uniform external field $E_{out} = E_0\, a_x$ in free space.

as shown in Figure, with free space on both sides of the slab and an external field

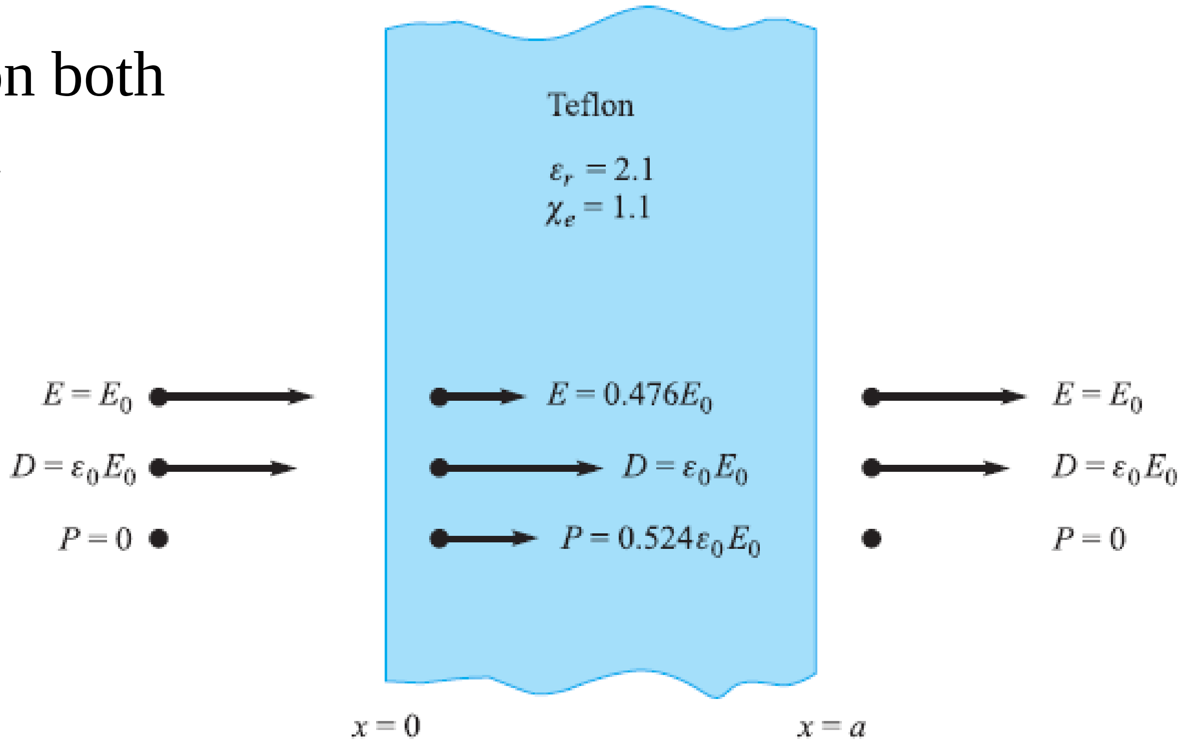
$$\mathbf{E}_{\text{out}} = E_0 \mathbf{a}_x$$

We also have

$$\mathbf{D}_{\text{out}} = \epsilon_0 E_0 \mathbf{a}_x$$

and

$$\mathbf{P}_{\text{out}} = 0.$$



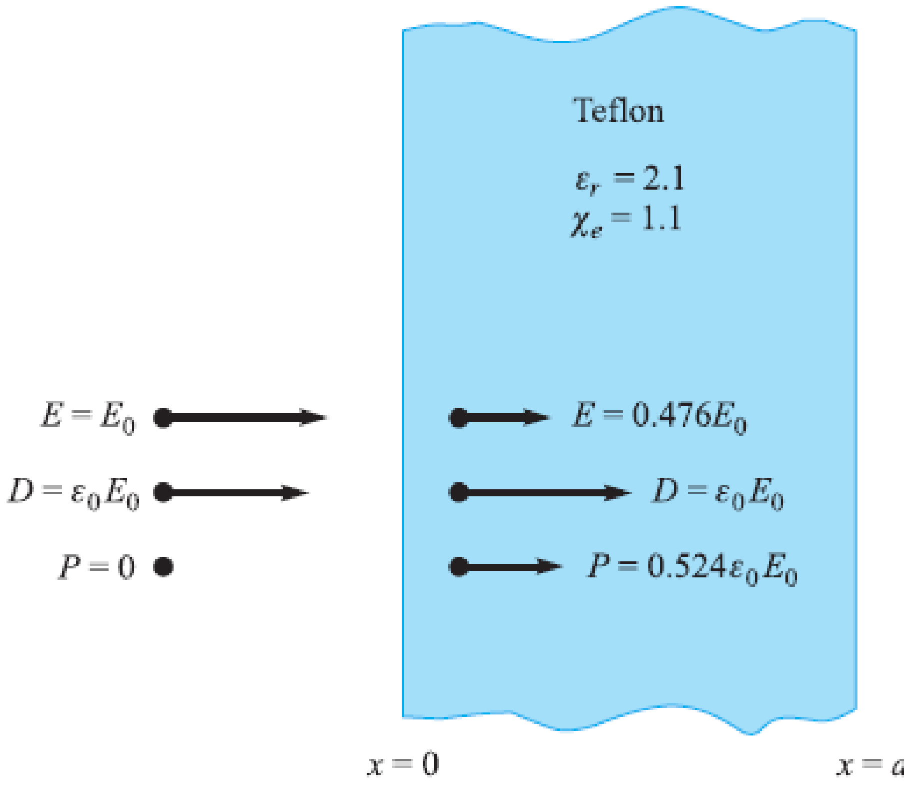
Boundary conditions for Dielectric materials

Inside, the continuity of D_N at the boundary allows us to find that $\mathbf{D}_{\text{in}} = \mathbf{D}_{\text{out}} = \varepsilon_0 E_0\, \mathbf{a}x$. This gives us

$$\mathbf{E}_{\text{in}} = \mathbf{D}_{\text{in}}/\epsilon = \epsilon_0 E_0 \mathbf{a}_x/(\epsilon_r \epsilon_0) = 0.476 E_0 \mathbf{a}_x$$

To get the polarization field in the dielectric, we use $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$ and obtain

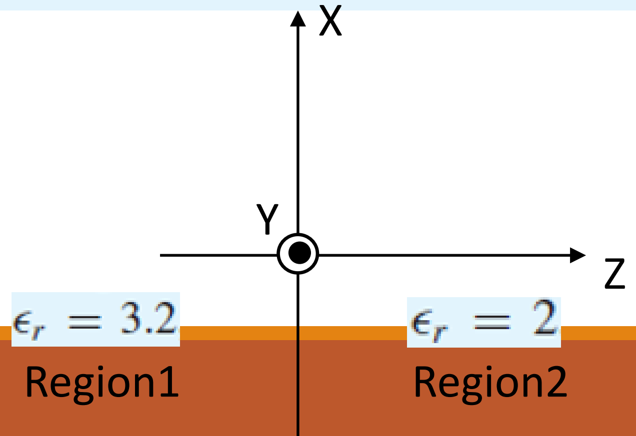
$$\begin{aligned}\mathbf{P}_{\text{in}} &= \mathbf{D}_{\text{in}} - \epsilon_0 \mathbf{E}_{\text{in}} \\ &= \epsilon_0 E_0 \mathbf{a}_x - 0.476 \epsilon_0 E_0 \mathbf{a}_x \\ &= 0.524 \epsilon_0 E_0 \mathbf{a}_x\end{aligned}$$



Boundary conditions for Dielectric materials

D5.9. Let Region 1 ($z < 0$) be composed of a uniform dielectric material for which $\epsilon_r = 3.2$, while Region 2 ($z > 0$) is characterized by $\epsilon_r = 2$. Let $\mathbf{D}_1 = -30\mathbf{a}_x + 50\mathbf{a}_y + 70\mathbf{a}_z$ nC/m^2 and find: (*a*) D_{N1}; (*b*) $\mathbf{D}_{t1}$; (*c*) $|D_{t1}|$; (*d*) $|D_1|$; (*e*) θ_1; (*f*) $\mathbf{P}_1$.

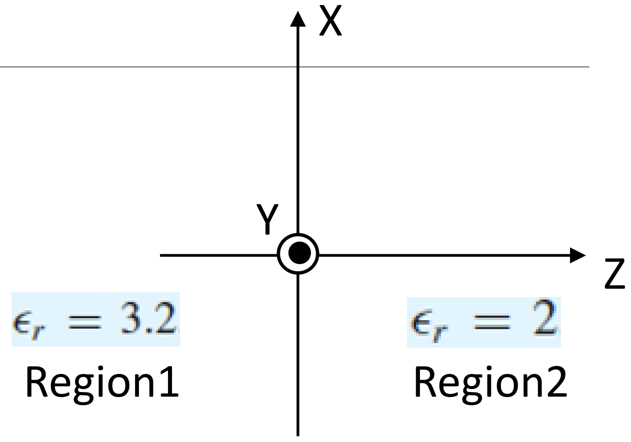
Ans. 70 nC/m^2; $-30\mathbf{a}_x + 50\mathbf{a}_y$ nC/m^2; 58.3 nC/m^2; 91.1 nC/m^2; 39.8°; $-20.6\mathbf{a}_x + 34.4\mathbf{a}_y + 48.1\mathbf{a}_z$ nC/m^2



Boundary conditions for Dielectric materials

The boundary at plane Z = 0, hence

$$\mathbf{D}_1 = \underbrace{-30\mathbf{a}_x + 50\mathbf{a}_y}_{D_{t1}} + \underbrace{70\mathbf{a}_z}_{D_{N1}}\ \mathrm{nC/m^2}$$

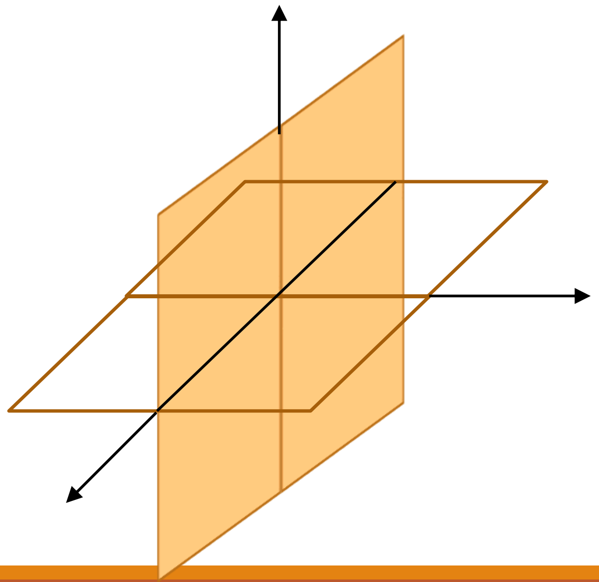


$$|D_{t1}| = \sqrt{D_x^2 + D_y^2} \qquad |D_1| = \sqrt{D_x^2 + D_y^2 + D_z^2}$$

$$\theta_1 = \cos^{-1}\left(\frac{D_{N1}}{D_1}\right)$$

$$P_1 = D_1 - \varepsilon_0 E_1$$

$$\because E_1 = D_1 / \varepsilon_0 \varepsilon_r \quad \Rightarrow \quad \therefore P_1 = D_1\left(1 - \frac{1}{\varepsilon_r}\right)$$



Boundary conditions for Dielectric materials

D5.10. Continue Problem D5.9 by finding: (*a*) $\mathbf{D}_{N2}$; (*b*) $\mathbf{D}_{t2}$; (*c*) $\mathbf{D}_2$; (*d*) $\mathbf{P}_2$; (*e*) θ_2.

Ans. $70\mathbf{a}_z$ nC/m^2; $-18.75\mathbf{a}_x + 31.25\mathbf{a}_y$ nC/m^2; $-18.75\mathbf{a}_x + 31.25\mathbf{a}_y + 70\mathbf{a}_z$ nC/m^2; $-9.38\mathbf{a}_x + 15.63\mathbf{a}_y + 35\mathbf{a}_z$ nC/m^2; 27.5°

Capacitance

The characteristic that all dielectric materials have in common, whether they are solid, liquid, or gas, and whether or not they are crystalline in nature, is their ability to store electric energy.

This storage takes place by means of a shift in the relative positions of the internal, bound positive and negative charges against the normal molecular and atomic forces.

A capacitor is a device that stores energy; energy thus stored can either be associated with accumulated charge or it can be related to the stored electric field.

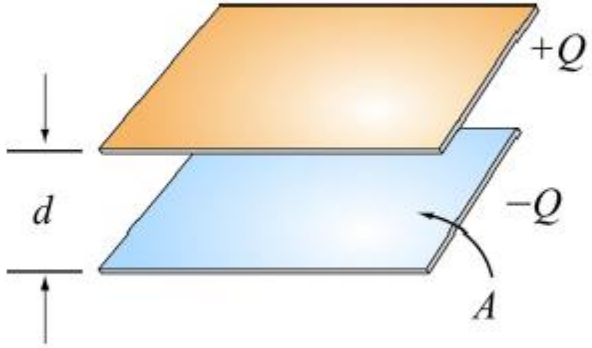
Capacitance

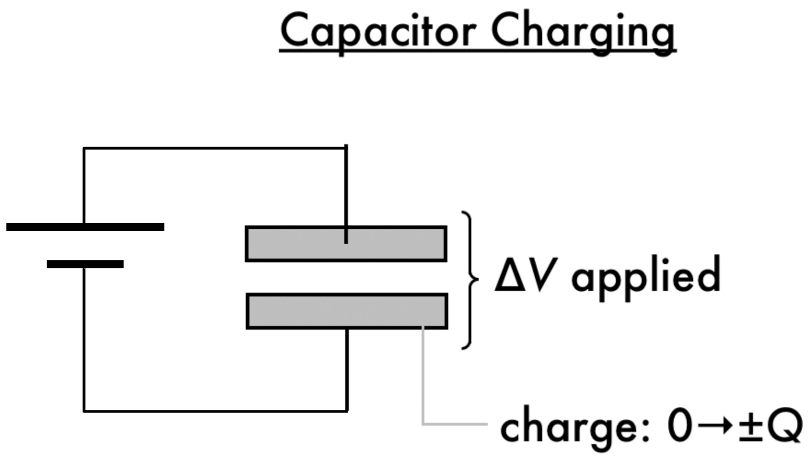
A capacitor is a device for storing electrical charge.

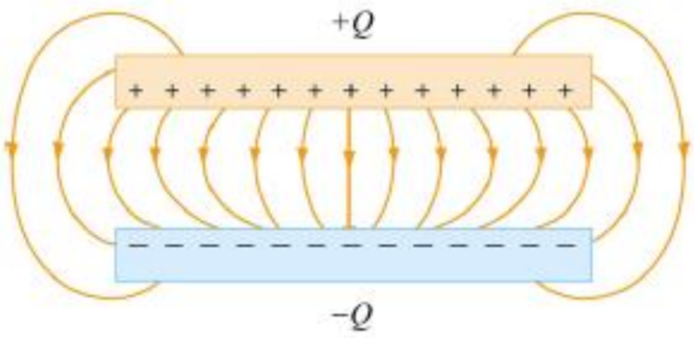
Capacitors consist of a pair of conducting plates separated by an insulating material (oil, paper, air).

The measure of the extent to which a capacitor can store charge is called Capacitance.

Capacitance is measured in farads F, or more usually microfarads μF or Pico farads pF.





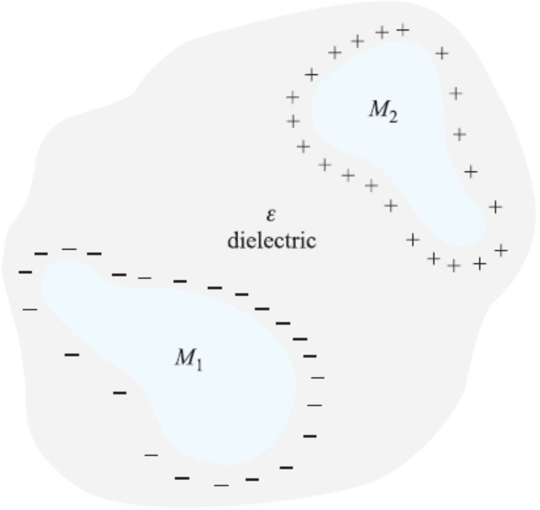


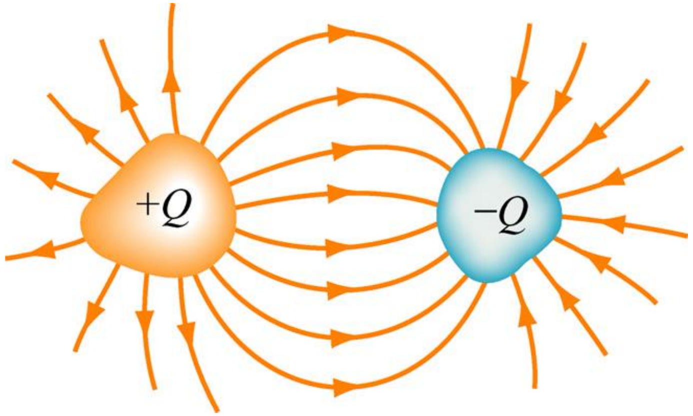
Capacitance

We define the *capacitance* of a two-conductor system as "the ratio of the magnitude of the total charge on either conductor to the magnitude of the potential difference between conductors".

$$C = \frac{Q}{V_0}$$ Farad

1 F = 1 farad = 1coulomb/volt





Capacitance

In general terms, we determine Q by a surface integral over the positive conductors, and we find V_0 by carrying a unit positive charge from the negative to the positive surface,

$$C = \frac{\oint_S \epsilon \mathbf{E} \cdot d\mathbf{S}}{-\int_-^+ \mathbf{E} \cdot d\mathbf{L}}$$

The capacitance is independent of the potential and total charge, for their ratio is constant.

The capacitance is a function only of the physical dimensions of the system of conductors and of the permittivity of the homogeneous dielectric.

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

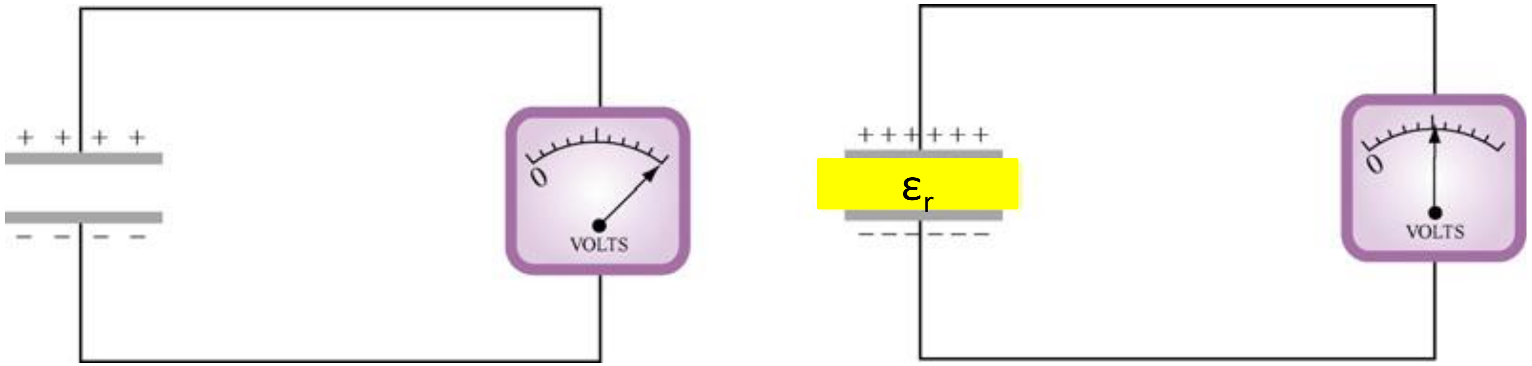
For parallel plate capacitor

$$C = \frac{2\pi \varepsilon_0 \varepsilon_r L}{\ln(b/a)}$$

For coaxial cable

Capacitance

Case 1: Dielectrics without Battery



- As shown in Figure, a battery with a potential difference $|\Delta V_0|$ is first connected to a capacitor C_0, which holds a charge $Q_0 = C_0|\Delta V_0|$. We then disconnect the battery, leaving Q_0 = const.
- insert a dielectric between the plates, while keeping the charge constant, experimentally it is found that

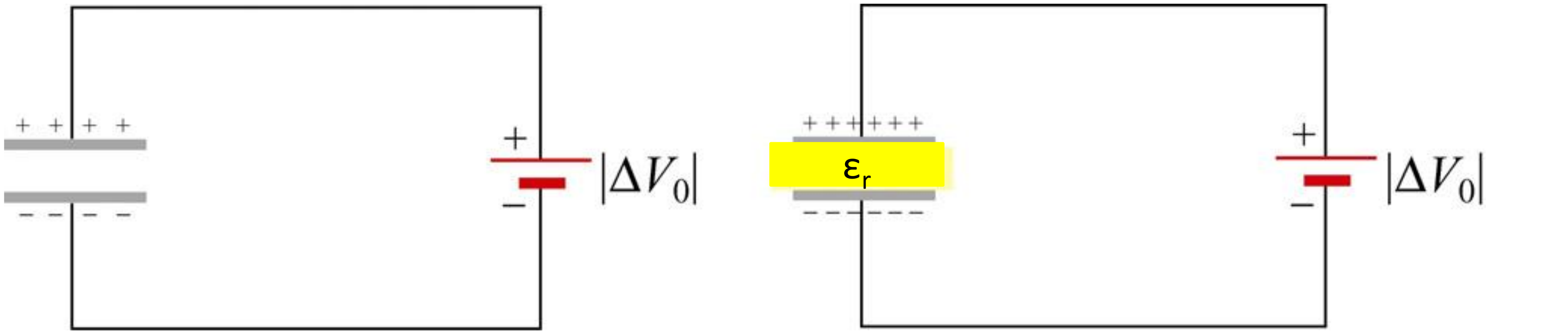
$$|\Delta V| = \frac{|\Delta V_0|}{\varepsilon_r}$$

- This implies that the capacitance is changed to

$$C = \frac{Q_0}{|\Delta V|} = \frac{Q_0}{|\Delta V_0|/\varepsilon_r} = \varepsilon_r C_0$$

Capacitance

Case 2: Dielectrics with Battery



- Consider a second case where a battery supplying a potential difference $|\Delta V_0|$ remains connected as the dielectric is inserted. Experimentally, it is found

$$Q = \varepsilon_r Q_0$$

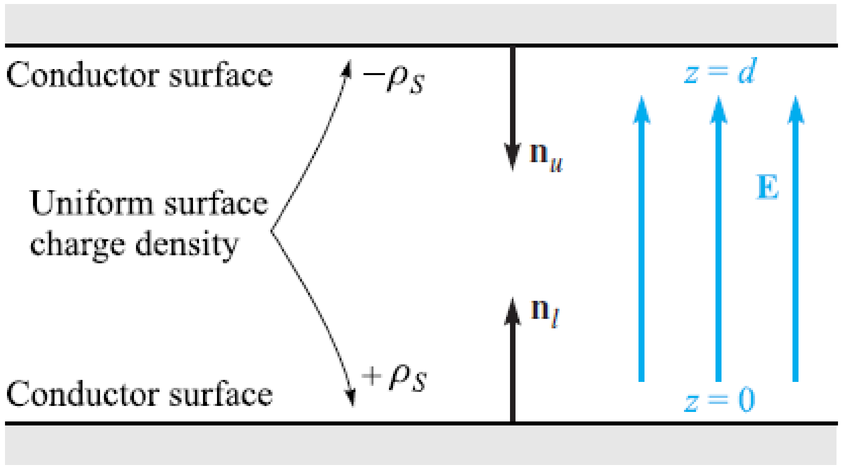
- where Q_0 is the charge on the plates in the absence of any dielectric.
- This implies that the capacitance is changed to

$$C = \frac{Q}{|\Delta V_0|} = \frac{\varepsilon_r Q_0}{|\Delta V_0|} = \varepsilon_r C_0$$

Capacitance

Parallel-plate Capacitor

Consider two metallic plates of equal area A separated by a distance d, as shown in the figure below. a uniform sheet of surface charge $\pm\rho_S$ on each conductor leads to the uniform field $\mathbf{E} = \frac{\rho_S}{\epsilon}\mathbf{a}_z$ and $\mathbf{D} = \rho_S \mathbf{a}_z$



Capacitance

Parallel-plate Capacitor

The potential difference between lower and upper planes is

$$V_0 = -\int_{upper}^{lower} E \cdot dL = -\int_{d}^{0} \frac{\rho_S}{\varepsilon} dz = \frac{\rho_S}{\varepsilon} d$$

$$\because Q = \rho_S A$$

$$V_0 = \frac{\rho_S}{\varepsilon} d$$

$$\therefore C = \frac{Q}{V_0} = \frac{\varepsilon A}{d}$$

Capacitance

Parallel-plate Capacitor

D6.1. Find the relative permittivity of the dielectric material present in a parallel-plate capacitor if: (*a*) $S = 0.12\ \text{m}^2$, $d = 80\ \mu\text{m}$, $V_0 = 12$ V, and the capacitor contains 1 μJ of energy; (*b*) the stored energy density is 100 J/m^3, $V_0 = 200$ V, and $d = 45\ \mu\text{m}$; (*c*) $E = 200$ kV/m and $\rho_S = 20\ \mu\text{C/m}^2$.

Ans. 1.05; 1.14; 11.3

Capacitance

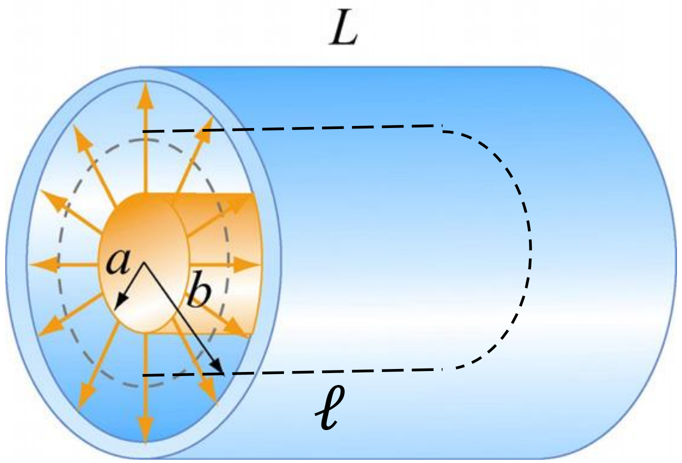
Cylindrical Capacitor

To calculate the capacitance, we first compute the electric field. Due to the cylindrical symmetry of the system, we choose our Gaussian surface to be a coaxial cylinder with length ℓ < L and radius r where $a < r < b$. Using Gauss's law, we have

$$\oiint_S E \cdot dA = Q_{encolsed} / \varepsilon$$

$$EA = E(2\pi r\ell) = \frac{\rho_l \ell}{\varepsilon}$$

$$\therefore E = \frac{\rho_l}{2\pi\varepsilon r}$$



Where ρ_l is the line charge density, and $\rho_l = \frac{Q}{L}$

Capacitance

Cylindrical Capacitor

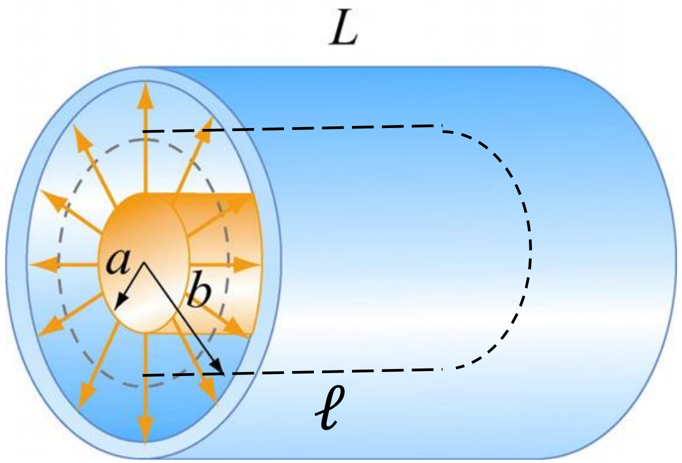
The potential difference is given by

$$\Delta V = V_a - V_b = -\int_b^a E_r \, dr$$

$$= -\frac{\rho_l}{2\pi\varepsilon}\int_b^a \frac{dr}{r} = \frac{\rho_l}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{\rho_l L}{\rho_l \ln(b/a)/2\pi\varepsilon}$$

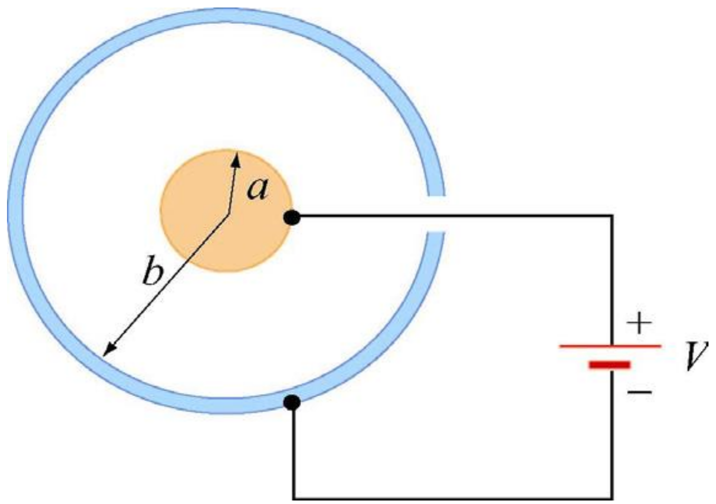
$$\therefore C = \frac{2\pi\varepsilon L}{\ln(b/a)}$$



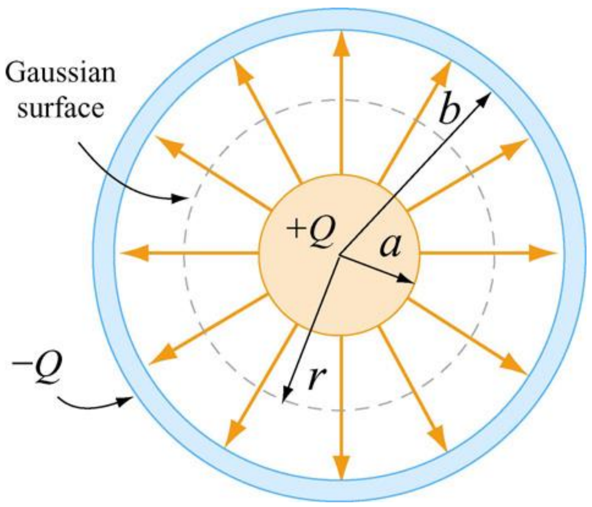
Capacitance

Spherical Capacitor

let's consider a spherical capacitor which consists of two concentric spherical shells of radii *a* and *b*, as shown in Figure



(a) spherical capacitor with two concentric spherical shells of radii *a* and *b*.



(b) Gaussian surface for calculating the electric field.

Capacitance

Spherical Capacitor

The electric field is non-vanishing only in the region a < r < b. Using Gauss's law, we obtain

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E_r A = E_r \left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

$$E_r = \frac{1}{4\pi\varepsilon_o} \frac{Q}{r^2}$$

Therefore, the potential difference between the two conducting shells is:

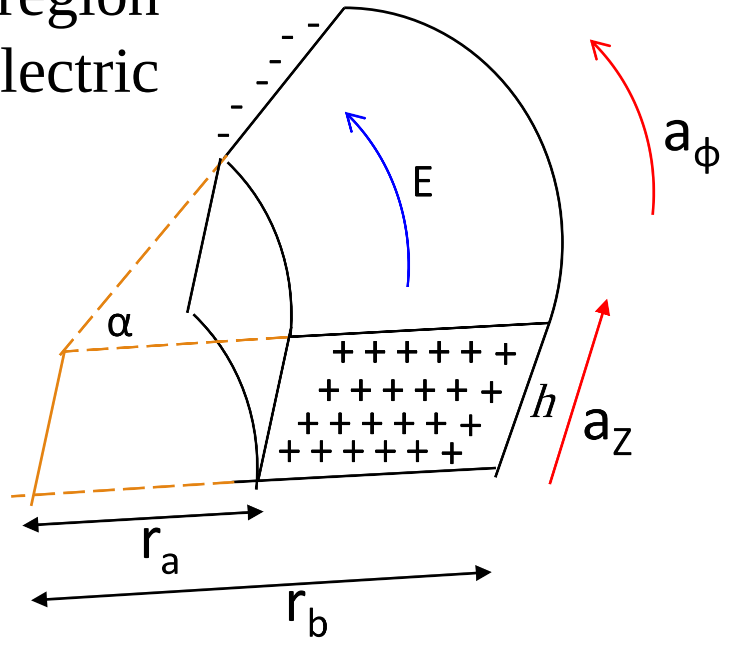
$$\Delta V = V_a - V_b = -\int_b^a E_r \, dr = -\frac{Q}{4\pi\varepsilon_0} \int_b^a \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

which gives

$$\boxed{C = \frac{Q}{|\Delta V|} = 4\pi\varepsilon_0 \left(\frac{ab}{b-a}\right)}$$

Capacitance

Find the capacitance shown in the figure, where the region between the plates is filled with a dielectric having a dielectric constant ε_r



Solution:

Assuming the voltage applied to the electrodes is V_0

$$V_0 = -\int_{-}^{+} E \cdot dl$$

Using the cylindrical coordinates system (r, φ, z), it is clear that the field is in the a_φ direction

$$V_0 = -\int_{\alpha}^{0} E\vec{a}_\phi \cdot d\vec{l}_\phi \qquad \text{Where} \quad dl_\phi = rd\phi$$

$$= -\int_{\alpha}^{0} E_\phi r d\phi = E_\phi r\alpha$$

$$\therefore E_\phi = V_0 / r\alpha \qquad , D_\phi = \varepsilon E_\phi = \varepsilon V_0 / r\alpha$$

Capacitance

From the boundary conditions for conductors

$$\rho_S = \left|\overrightarrow{D}_n\right| = \varepsilon V_0 / r\alpha$$

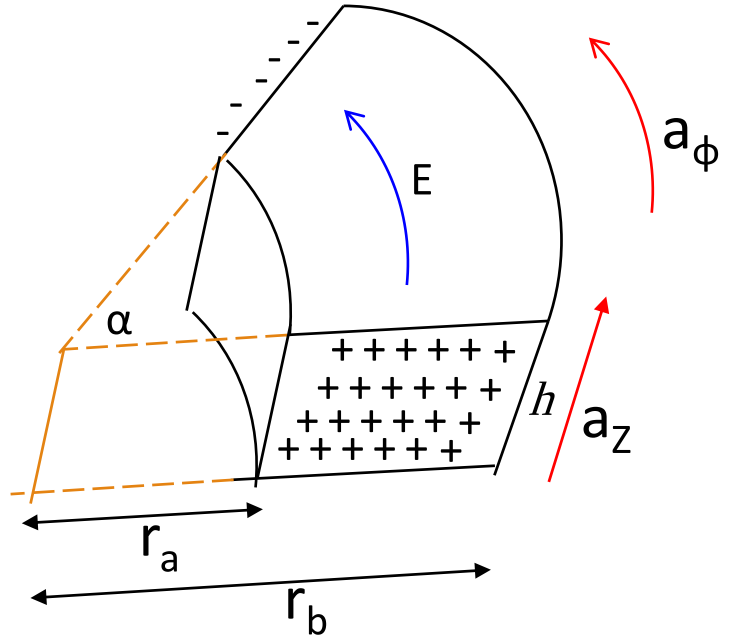
$\therefore$ The total charge on the plate

$$Q = \iint \rho_S ds\,, \quad ds = drdz$$

$$Q = \int_{r_a}^{r_b}\int_0^h (\varepsilon V_0 / r\alpha) drdz$$

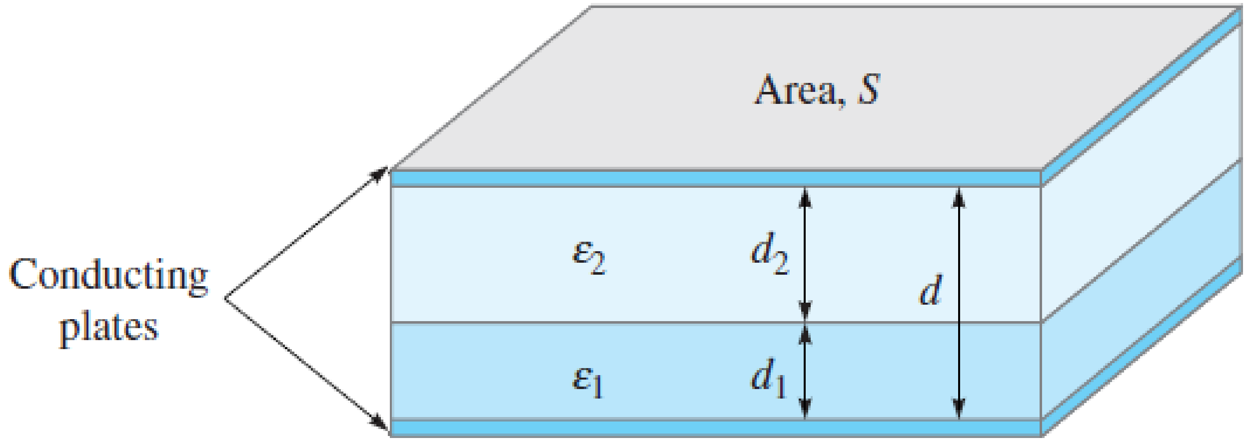
$$= \int_{r_a}^{r_b} \left(\frac{\varepsilon V_0 h}{\alpha}\right)\frac{dr}{r} = \frac{\varepsilon V_0 h}{\alpha}\ln\left(\frac{r_b}{r_a}\right)$$

$$C = \frac{Q}{V_0} = \frac{\varepsilon h}{\alpha}\ln\left(\frac{r_b}{r_a}\right) \qquad \therefore C = \frac{\varepsilon_0\varepsilon_r h}{\alpha}\ln\left(\frac{r_b}{r_a}\right)$$



Multiple-Dielectric Capacitors

Let's consider the two parallel-plate capacitor having two dielectric materials ε_1 and ε_2 , as shown in the figure below, and their thickness are d_1 and d_2, respectively.

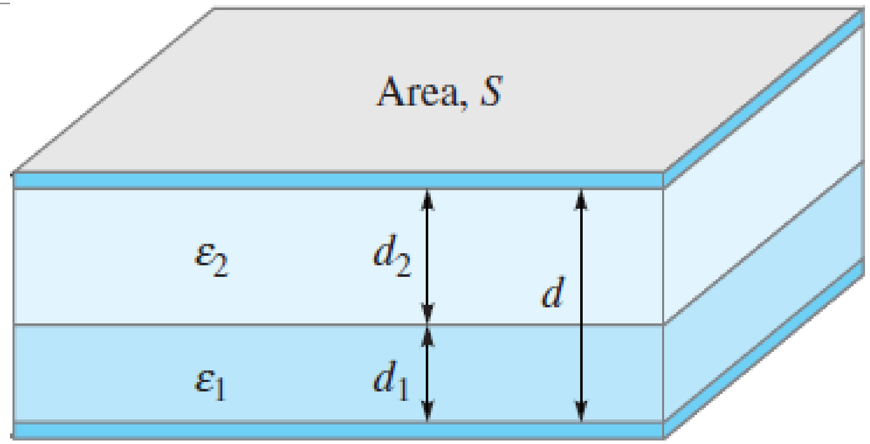


parallel-plate capacitor containing two dielectrics with the dielectric interface parallel to the conducting plates.

Multiple-Dielectric Capacitors

Suppose we assume a potential difference V_0 between the plates. The electric field intensities in the two regions, E_2 and E_1, are both uniform,

$$V_0 = E_1 d_1 + E_2 d_2$$



At the dielectric interface, E is normal, and from the boundary conditions

$$D_{N1} = D_{N2} \Rightarrow \varepsilon_1 E_1 = \varepsilon_2 E_2$$

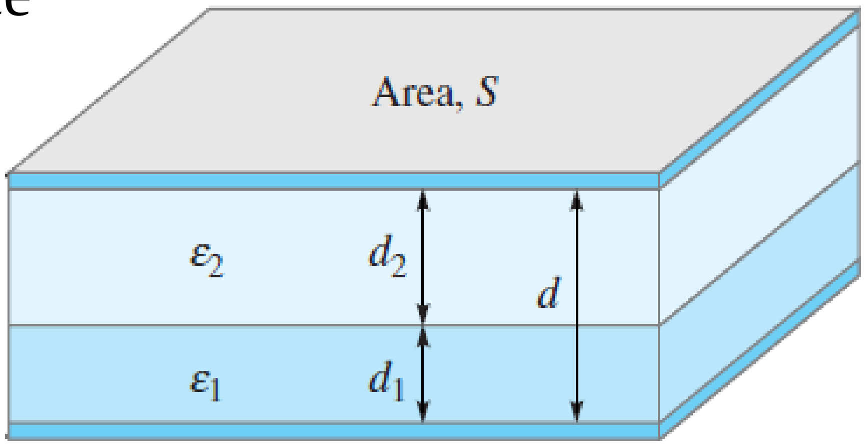
$$\begin{aligned} V_0 &= E_1 d_1 + E_2 d_2 \\ &= E_1 d_1 + \left[\left(\varepsilon_1/\varepsilon_2\right)E_1\right]d_2 \end{aligned}$$

$$\therefore E_1 = \frac{V_0}{d_1 + \left(\varepsilon_1/\varepsilon_2\right)d_2}$$

Multiple-Dielectric Capacitors

From the boundary conditions for conductors, the surface charge density on the lower plate has the magnitude

$$\rho_{S1} = D_1 = \varepsilon_1 E_1 = \frac{V_0}{(d_1/\varepsilon_1) + (d_2/\varepsilon_2)}$$



Because $D_1 = D_2$, the magnitude of the surface charge is the same on each plate. The capacitance is then

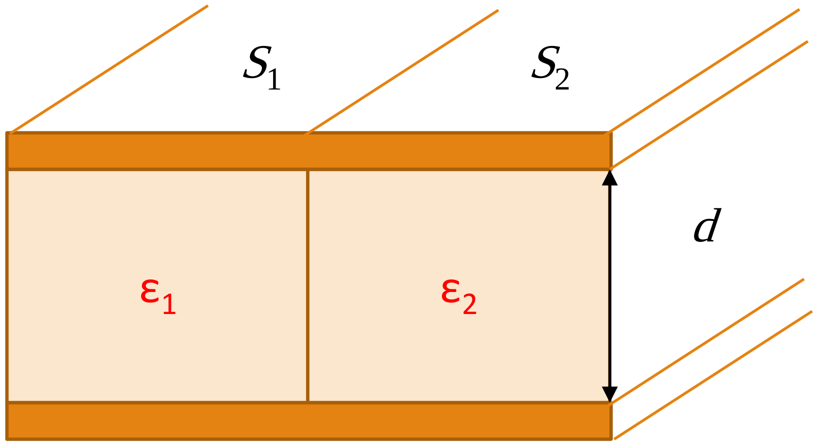
$$C = \frac{Q}{V_0} = \frac{\rho_S S}{V_0} = \frac{1}{(d_1/S\varepsilon_1) + (d_2/S\varepsilon_2)}$$

$$\therefore C = \frac{1}{(1/C_1) + (1/C_2)}$$

Multiple-Dielectric Capacitors

If the dielectric boundary were placed *normal* to the two conducting plates and the dielectrics occupied areas of S_1 and S_2, as shown in the figure

then an assumed potential difference V_0 would produce field strengths $E_1 = E_2 = V_0/d$



$$C = \frac{Q}{V_0} = \frac{\rho_{S1}S_1 + \rho_{S2}S_2}{Ed} = \frac{\varepsilon_1 ES_1 + \varepsilon_2 ES_2}{Ed}$$

$$C = \frac{\varepsilon_1 S_1 + \varepsilon_2 S_2}{d}$$

$$\therefore C = C_1 + C_2$$

Multiple-Dielectric Capacitors

D6.2. Determine the capacitance of: (*a*) a 1-ft length of 35B/U coaxial cable, which has an inner conductor 0.1045 in. in diameter, a polyethylene dielectric ($\epsilon_r = 2.26$ from Table C.1), and an outer conductor that has an inner diameter of 0.680 in.; (*b*) a conducting sphere of radius 2.5 mm, covered with a polyethylene layer 2 mm thick, surrounded by a conducting sphere of radius 4.5 mm; (*c*) two rectangular conducting plates, 1 cm by 4 cm, with negligible thickness, between which are three sheets of dielectric, each 1 cm by 4 cm, and 0.1 mm thick, having dielectric constants of 1.5, 2.5, and 6.

Ans. 20.5 pF; 1.41 pF; 28.7 pF


Thank you